Expanded Tertiary Tree of Primitive Pythagorean Triples: MIDDLE derived BRANCH

Trunk

TRUNK	PPT	a	Δa	b	Δb	c	Δc	ΔEVEN a or b - EVEN a or b	$\sum a+b+c = r_{next}$	r	Δr	$\Delta r \div 2$	$r^2 \div 4$	$\Delta r^2 \div 4$	s	t	Δt	$2c^2 = c2c = t_x$	$2c^2-1 = s_x$	$\sqrt{(2c^2-1)} = \Delta s_x$ or Δt_x	A	A ÷ 6	A/6 + A_{next}/6 = $r^2/4_{next}$	P	ΔP	$\Delta P \div 2$	f	$U \div c = p$	Δp	$\Delta p \div 4$
1	**3-4-5**	3		4		5			12	**2**			**1**		1	2		50 = 5 x 10	49 = 7^2	**7**	6	1	**36**	12			1	1		

1st-9th MIDDLE Tertiary Branches

MIDDLE TERTIARY BRANCH	PPT	a	Δa	b	Δb	c	Δc	ΔEVEN a or b - EVEN a or b	$\sum a+b+c = r_{next}$	r	Δr	$\Delta r \div 2$	$r^2 \div 4$	$\Delta r^2 \div 4$	s = $(a+c)$ previous	t = $(b+c)$ previous	Δt	$2c^2 = c2c = t_x$	$2c^2-1 = s_x$	$\sqrt{(2c^2-1)} = \Delta s_x$ or Δt_x	A	A ÷ 6	A/6 + A_{next}/6 = $r^2/4_{next}$	P	ΔP	$\Delta P \div 2$	f	$U \div c = p$	Δp	$\Delta p \div 4$
1	**20-21-29**	20		21		29		**16 = 4^2**	**70**	12	**10**	5	**36 = 6^2**	**5**	8	9	**7**	**1682 = 29×58**	**1681 = 41^2**	**41**	210	35	**1225**	70	**58**	**29**	1	**5**	**4**	**1**
2	**119-120-169**	119		120		**169**		**100 = 10^2**	**408**	70	**58**	29	**1225 = 35^2**	**29**	49	50	**41**	**57122 = 169×338**	**57121 = 239^2**	**239**	7140	1190	**41616**	408	**338**	**169**	1	**29**	**24**	**6**
3	**696-697-985**	696		697		985		**576 = 24^2**	**2378**	408	**338**	169	**41616 = 204^2**	**169**	288	289	**239**	**1940450 = 985×1970**	**1940449 = 1393^2**	**1393**	242556	40426	**141321**	2378	**1970**	**985**	1	**169**	**140**	**35**
4	**4059-4060-5741**	4059		4060		5741		**3364 = 58^2**	**13860**	2378	**1970**	985	**141321 = 1189^2**	**985**	1681	1682	**1393**	**65918162 = 5741x11482**	**65918161 = 8119^2**	**8119**	8239770	1373295	**48029900**	13860	**11482**	**5741**	1	**985**	**816**	**204**
5	23660-23661-33461	23660		23661		33461		**19600 = 140^2**	**80782**	13860	**11482**	5741	**48029900 = 6930^2**	**5741**	9800	9801	**8119**	**33461 x 66922**	**2239277041 = 47321^2**	**477321**	279909630	46651605	**163143281**	80782	**66922**	**33461**	1	**5741**	**4756**	**1189**
6	137903-137904-195025	137903		137904		195025		**114244 = 338^2**	**470832**	80782	**66922**	33461	**1631432881 = 40391^2**	**33461**	57121	57122	**47321**	**195025 x 390050**	**76069501249 = 275807^2**	**275807**	9508687656	1584781276	**55420693056**	470832	**390050**	**195025**	1	**33461**	**27720**	**6930**
7	803760-803761-1136689	803760		803761		1136689		**665856 = 816^2**	**2744210**	470832	**390050**	195025	**55420693 056 = 235416^2**	**195025**	332928	332929	**275807**	**1136689 x 2273378**	**2584123765441 = 1607521^2**	**1607521**	323015470680	53835911780	**1882672131025**	2744210	**2273378**	**1136689**	1	**195025**	**161564**	**40391**
8	4684659-4684660-6625109	4684659		4684660		6625109		**3880900 = 1970^2**	**15994428**	2744210	**2273378**	1136689	**1882 672131025 = 137210^2**	**1136689**	1940449	1940450	**1607521**	**6625109 x 13250218**	**93693192^2**	**9369319**	10973017315470	1828836219245	***see $r^2 \div 4$***	15994428	**13250218**	**6625109**	1	**1136689**	**941664**	**235416**
9	27304196-27304197-38613965	27304196		27304197		38613965		**22619536 = 4756^2**	**93222358**	15994428	**13250218**	6625109	**63955 431761796 = 799714^2**	**6625109**	11309768	11309769	**9369319**	**38613965 x 77227930**			372759573255306	32126595542551		93222358	**77227930**	**38613965**	1	**6625109**	**5488420**	**1372105**
Notes	$\sum a+b+c = r_{next}$	$a + c = s_{next}$ $a = r + s$ $a = c - t$		$b + c = t_{next}$ $b = r + t$ $b = c - s$		$c = p_{next}$ $c = 2r + p$ $c = r + s + t$ $c = a + t$ $c = b + t$ $c = a + b - r$ $c = U/p$		Δ of EVEN a or b from the NEXT EVEN a or b value = 4—25—144—841— …	$\sum a+b+c = r_{next}$	The $\Delta r = \sum s + t_{next} = \sum s_{next} + t$ The $\Delta r/2$ = 5—29—169—… and = the p & $\Delta P/2$ $r = \sqrt{2st}$ $r = (a + b) - c$.			Dividing r^2 by 4 is the same as ÷ it by the r^2 of 3-4-5. The Δ in $r^2/4$ also generates the same 5—29—169…sequence.		$r^2/2 = st$. $s_{next} = a + c$ $t_{next} = b + c$ The s & t are a hybrid of the larger s & t from the UPPER & LOWER Branches, e.i. 8 comes from the UPPER and 9 the LOWER. The Δ in either s or t as one moves down the columns is reflected in $\sqrt{(2c^2-1)}$ column. Its square = $2c^2-1$ column and the s value is found in x number of rows below the s. The $2c^2 = c2c = t_x$ shows how c x $2c$ equals the t value some x number of rows below t.						The $\sum$ of A/6 + A_{next}/6 = $r^2/4_{next}$ $A = (Pr)/4 = bh/2$			$P = \sum a+b+c = (4A)/r = 2c + r = r_{next}$			All f = 1	$p = c - 2r$ $p_{next} = c$ $\Delta p \div 4 = r_{previous} / 2$ On the matrix, a 45° diagonal from p on the AXIS is a COMMON DIAGONAL to all three TERTIARY MEMBERS of a given Branch. This unites them.		

Summary

Finding the NEXT MIDDLE is easy! Since we do not know: $a\uparrow$, $b\uparrow$ or $c\uparrow$ to generate the NEXT, e.i. to **20—21—29** to **119—120—169** MIDDLE PPT, we will move to the second method for finding the NEXT MIDDLE PPT.

Because $\sum a+b+c = \sum = r_{next} = {}^*P$ we have for the ***20—21—29***: (20+21+29) =70 = $r_{next} = P$, with P=70. We also know that $r\uparrow$ as $\Delta r/2$, or $\Delta r^2/4$, follows the 5-29-169-985-…sequence of the c-value, so r + (2x29) = 12 + 58 = 70 = r_{next}. *Then* $r^2/2 = st = 70^2/2$ = 2450 & 2450/49=50.

Knowing $s_{next} = (a + c)_{previous}$, s_{next} = 20 + 29 = 49, then t_{next} = 50, confirmed by knowing that $\sqrt{(2c^2-1)} = \Delta s_x$ or Δt_x = 41, t_{next} = 9 + 41 = 50, as does $b + c = t_{next}$ = 21 + 29 = 50, we can easily calculate a, b & c:

$a = r + s$ = 70 + 49 = 119 $b = r + t$ = 70 + 50 = 120 $c = r + s + t$ = 70 + 49 + 50 = 169 to give the **119—120—169** NEXT MIDDLE PPT.

As $P\uparrow$ by the $\Delta P/2$ = the 5-29-169-985-…sequence of the c-value, we add (2x169) + 70 = 408, confirming $P = a + b + c$ = 119 + 120 + 169 = 408. Starting with the P of the **20—21—29,** $P = 2c + r$ = (2x29) + 12 = 70 & $^*P = r_{next}$ = 70 = the r-value of the **119—120—169 PPT.**

From $f = t - s = b - a$ = 50 - 49 = 120 - 119 = 1, we also know $f\uparrow$ by 0 (constant). $p\uparrow$. $c = p_{next}$ is another confirming pattern, as the c = 29 of the **20—21—29** = the p_{next}, *i.e. the* p=29, $p = c - 2r = 169 - 140$, *of the* **119—120—169 PPT. All Tertiary Branch Clusters have the same** ***p*****-value!**

Table 3c

Key: **PPT=Primitive Pythagorean Triple**; **r=even #** such that $r^2/2 = st$ where **s,t** are Factor Pairs; **A=Area; 4A=4Area; 8A=8Area; $f = b - a = t - s$** & **$f^2 = (b-a)^2$**, as **$a^2 + b^2 = c^2$ = 4A + f^2 = (8A + f^2) - 4A** & **$U/c = p$. $U = s^2 + t^2$ $A = Pr/4$** & **$P = 4A/r = 2c + r$**, whereas **$c = 2r + p$**.
The Tree of Pythagorean Triples branches from the **3-4-5 PPT Trunk** first into a 3-part main branch, each of which further branches into 2nd, 3rd, 4th, …, **Tertiary Branches**. Each **Tertiary** follows the lead **f**-value of its predecessor, but is actually formed as an intermediary to the Upper and Lower branches of which it is a part. All **PPTs** — with no repeats — are to be found. ***Pythagoras*** first discovered the **UPPER Branch** sequence, ***Plato*** (a century later) discovered the **LOWER Branch** sequence. The **MIDDLE Branch** sequence follows as an intermediary, hybrid sequence of the **UPPER** and **LOWER**, plus some amazing ***Number Pattern Sequences*** (***NPS***) all to itself.
Using the ***Expanded Dickson Method*** on the **BBS-ISL Matrix**, every **PPT Branch** is accounted for by the previous **Branch**. This is done by enlisting the ***r,s,t,A,4A,8A,f*** associated values as seen in the **Table 2** series . All these values are derived directly from the respective **PTT** by both algebra and geometry. Now, in **Table 3c,** we look at the overall ***NPS*** of just one **Branch** sequence: here we are looking at the **MIDDLE derived Branches**. By parsing out the differences, **Δ**, in the individual ***a, b, c, r, r², s, t, 2c², √², A, P, ƒ & p*** values one can see the incredible way the fundamental **ISL** number sequence — a sequence that informs the entire **BBS-ISL Matrix** — certainly comes into play here to form a consistent ***NPS*** link from and to each and every **PPT** on its **Branch**.