Table 3b

Expanded Tertiary Tree of Primitive Pythagorean Triples: LOWER *Plato* BRANCH

TRUNK	**PPT**	*a*	*Δa*	*b*	*Δb*	*c*	*Δc*	*ΔEVEN a or b - EVEN a or b*	*(Σa+b+c)÷ a/2 =* r_{next}	*r*	*Δr*	*Δr ÷ 2*	r^2 *÷ 4*	*Δ*r^2 *÷ 4*	*s*	*t*	*Δt*	*2*c^2	*Δt ÷ 8*	**A**	**A ÷ 6**	**ΔA ÷ 6**	*P*	*ΔP*	**Δ ΔP**	*ƒ*	*Δƒ*	**Δ *ƒ*- *p***	***U ÷ c = p***	**Δ p**	**Δp ÷ 4**
	Trunk																														
1	**4-3–5**	4		3		5			**6**	2			1		2	1		50		6	1		12			**1**		**0**	**1**		
	1st-14th LOWER Tertiary Branches																														
LOWER TERTIARY BRANCH	**PPT**	*a*	*Δa*	*b*	*Δb*		*Δc*	*ΔEVEN a or b - EVEN a or b*	*(Σa+b+c)÷ a/2 =* r_{next}	*r*	*Δr*	*Δr ÷ 2*	r^2 *÷ 4*	*Δ*r^2 *÷ 4*	*s*	*t = (a+c) previous*	*Δt*	*2*c^2	*Δt ÷ 8*	**A**	**A ÷ 6**	**ΔA ÷ 6**	*P*	*ΔP*	**Δ ΔP**	*ƒ*	*Δƒ*	**Δ *ƒ*- *p***	***U ÷ c = p***	**Δ p**	**Δp ÷ 4**
1	**8-15-17**	8	**4**	15	**12**	17	**12**	**4**	**10**	6	**4**	2	**$9=3^2$**	**8**	2	9	**8**	578	**1**	60	10	**9**	40	28	**16**	7	**6**	**2**	**5**	**4**	**1**
2	**12-35-37**	12	**4**	35	**20**	37	**20**	**4**	**14**	10	**4**	2	**$25=5^2$**	**16**	2	25	**16**	2738	**2**	210	35	**25**	84	44	**16**	23	**16**	**6**	**17**	**12**	**3**
3	**16-63-65**	16	**4**	63	**28**	65	**28**	**4**	**18**	14	**4**	2	**$49=7^2$**	**24**	2	49	**24**	8450	**3**	504	84	**49**	144	60	**16**	47	**24**	**10**	**37**	**20**	**5**
4	**20-99-101**	20	**4**	99	**36**	101	**36**	**4**	**22**	18	**4**	2	**$81=9^2$**	**32**	2	81	**32**	2402	**4**	990	165	**81**	220	76	**16**	79	**32**	**14**	**65**	**28**	**7**
5	**24-143-145**	24	**4**	143	**44**	145	**44**	**4**	**26**	22	**4**	2	**$121=11^2$**	**40**	2	121	**40**	42050	**5**	1716	286	**121**	312	92	**16**	119	**40**	**18**	**101**	**36**	**9**
6	**28-195-197**	28	**4**	195	**52**	197	**52**	**4**	**30**	26	**4**	2	**$169=13^2$**	**48**	2	169	**48**	77618	**6**	2730	455	**169**	420	108	**16**	167	**48**	**22**	**145**	**44**	**11**
7	**32-255-257**	32	**4**	255	**60**	257	**60**	**4**	**34**	30	**4**	2	**$225=15^2$**	**56**	2	225	**56**	132098	**7**	4080	680	**225**	544	124	**16**	223	**56**	**26**	**197**	**52**	**13**
8	**36-323-325**	36	**4**	323	**68**	325	**68**	**4**	**38**	34	**4**	2	**$289=17^2$**	**64**	2	289	**64**	211250	**8**	5814	969	**289**	684	140	**16**	287	**64**	**30**	**257**	**60**	**15**
9	**40-399-401**	40	**4**	399	**76**	401	**76**	**4**	**42**	38	**4**	2	**$361=19^2$**	**72**	2	361	**72**	321602	**9**	7980	1330	**361**	840	156	**16**	359	**72**	**34**	**325**	**68**	**17**
10	**44-483-485**	44	**4**	483	**84**	485	**84**	**4**	**46**	42	**4**	2	**$441=21^2$**	**80**	2	441	**80**	470450	**10**	10626	1771	**441**	1012	172	**16**	439	**80**	**38**	**401**	**76**	**19**
11	**48-575-577**	48	**4**	575	**92**	577	**92**	**4**	**50**	46	**4**	2	**$529=23^2$**	**88**	2	529	**88**	665858	**11**	13800	2300	**529**	1200	188	**16**	527	**88**	**42**	**485**	**84**	**21**
12	**52-675-677**	52	**4**	675	**100**	677	**100**	**4**	**54**	50	**4**	2	**$625=25^2$**	**96**	2	625	**96**	916658	**12**	17550	2925	**625**	1404	204	**16**	623	**96**	**46**	**577**	**92**	**23**
13	**56-783-785**	56	**4**	783	**108**	785	**108**	**4**	**58**	54	**4**	2	**$729=27^2$**	**104**	2	729	**104**	1232450	**13**	21924	3654	**729**	1624	220	**16**	727	**104**	**50**	**677**	**100**	**25**
14	**60-899-901**	60	**4**	899	**116**	901	**116**	**4**	**62**	58	**4**	2	**$841=29^2$**	**112**	2	841	**112**	1623602	**14**	26970	4495	**841**	1860	236	**16**	839	**112**	**54**	**785**	**108**	**27**
Notes	$\sum a+b+c = r_{next}$	$a + c = s_{next}$ $a = r + s$ $a = c - t$	$a\uparrow = 4$	$b + c = t_{next}$ $b = r + t$ $b = c - s$	$b\uparrow = 8 + \sum\Delta$	$c = p_{next}$ $c = 2r + p$ $c = r + s + t$ $c = a + t$ $c = b + t$ $c = a + b - r$ $c = U/p$	$c\uparrow = 8 + \sum\Delta$	Δ of EVEN *a* or *b* from the NEXT EVEN *a* or *b* value = 8+ ∑Δ	$P = \sum a+b+c = (4A)/r = 2c + r$ $P/a = r_{next}$	The $\Delta r = \sum s + t_{next} = \sum s_{next} + t$ The $\Delta r/2$ = 5—29—169—… and = the *p* & *ΔP/2* $r = \sqrt{2st}$ $r = (a + b) - c$. $r\uparrow = 4$			Dividing r^2 by 4 is the same as ÷ it by the r^2 of 3-4-5. The Δ in $r^2/4 = 8 + \sum\Delta$			$r^2/2 = st$. $s_{next} = a + c$ $t_{next} = b + c$ s = constant 2 $t\uparrow = 8 + \sum\Delta$ and when ÷ by 8 = 1—2—3—4—…. sequence that defines the AXIS of the BBS-ISL MATRIX (BIM)				The ∑ of A/6 - $A_{next}/6 = r^2/4_{next}$ $A = (Pr)/4 = bh/2$ When ΔA ÷ 6 = $1^2—3^2—5^2—7^2—9^2$— sequence that defines the BBS-ISL MATRIX (BIM)			$P = \sum a+b+c = (4A)/r = 2c + r$ $P/(a/2) = r_{next}$ The *Δ* in *P* is the accumulated *∑+ 16*			The Δ in *ƒ* is the accumulated ∑+ 8 The Δ between *ƒ* and p is the accumulated ∑+ 4			$p = c - 2r$ $p_{next} = c$ $\Delta p \div 4 = r_{previous}/2$ On the matrix, a 45° diagonal from *p* on the AXIS is a COMMON DIAGONAL to all three TERTIARY MEMBERS of a given Branch. This unites them.		

Summary

Finding the NEXT LOWER is easy! Since we know: $a\uparrow = 4$ $b\uparrow$ and $c\uparrow = \sum\Delta + 8$, one can simply add these differences to the previous to generate the NEXT, e.i. to **8—15—17** add 4 to *a*=12, *add* 8+12=20 to *b*=35 & 8+12=12 to *c*=37 to give the **12—35—37** NEXT LOWER PPT.

Because $(\sum a+b+c)/(a/2) = \sum/(a/2) = r_{next} = {}^*P/a$, we have for the ***8—15—17*** : (8+15+17)/(8/2) = 40/4 = 10 = $r_{next} = P/(a/2)$, with *P*=40. We also know that $r\uparrow$ by 4, so $r+4=6+4=10=r_{next}$. *Then* $r^2/2 = st = 10^2/2 = 50$.

Knowing *s*=constant 2, therefore t_{next} = 25, confirmed by knowing that $\sqrt{t_{next}}$= the 1^2-3^2-5^2-7^2-9^2… ISL sequence, and that $a + c = t_{next}$ = 8 + 17 = 25, we can easily calculate *a, b* & *c*:

$a = r + s$ = 10 + 2 = 12 $b = r + t$ = 10 + 25 = 35 $c = r + s + t$ = 10+2+25=37 to give the **12—35—37** NEXT UPPER PPT.

As $P\uparrow$ by the ∑Δ + 16, we add 28 + 16 = 44 to 40 = 84, confirming $P = a + b + c$ = 12 + 35 + 37 = 84, with the starting *P* of the **8—15—17,** $P = 2c + r$ = (2x17) + 6 = 40 & $^*P/(a/2) = r_{next}$= 40/(8/2) = 10 = the *r*-value of the **12—35—37 PPT.**

From $ƒ = t - s = b - a$ = 25 - 2 = 35 - 12 = 23, we also know $ƒ\uparrow$ by the ∑Δ + 8 sequence, as does the $p\uparrow$. $c = p_{next}$ is another confirming pattern, as the *c* = 17 of the **5—12—13** = the p_{next}, *i.e. the p=17 of* the **12—35—37 PPT. All Tertiary Branch Clusters have the same *p*-value!**

Table 3b

Key: **PPT=Primitive Pythagorean Triple**; ***r*=even #** such that $r^2/2 = st$ where ***s,t*** are Factor Pairs; **A=Area; 4A=4Area; 8A=8Area;** $f = b - a = t - s$ & $f^2 = (b-a)^2$, as $a^2 + b^2 = c^2 = 4A + f^2 = (8A + f^2) - 4A$ & $U/c = p$. $U = s^2 + t^2$ $A = Pr/4$ & $P = 4A/r = 2c + r$, whereas $c = 2r + p$.
The Tree of Pythagorean Triples branches from the **3-4-5 PPT Trunk** first into a 3-part main branch, each of which further branches into 2nd, 3rd, 4th, …, **Tertiary Branches**. Each **Tertiary** follows the lead ***f***-value of its predecessor, but is actually formed as an intermediary to the Upper and Lower branches of which it is a part. All **PPT**s — with no repeats — are to be found. ***Pythagoras*** first discovered the **UPPER Branch** sequence, ***Plato*** (a century later) discovered the **LOWER Branch** sequence. The **MIDDLE Branch** sequence follows as an intermediary, hybrid sequence of the **UPPER** and **LOWER**, plus some amazing ***Number Pattern Sequences*** (***NPS***) all to itself.
Using the ***Expanded Dickson Method*** on the **BBS-ISL Matrix**, every **PPT Branch** is accounted for by the previous **Branch**. This is done by enlisting the ***r,s,t,A,4A,8A,f*** associated values as seen in the **Table 2** series . All these values are derived directly from the respective **PTT** by both algebra and geometry. Now, in **Table 3c,** we look at the overall ***NPS*** of just one **Branch** sequence: here we are looking at the **MIDDLE derived Branches**. By parsing out the differences, **Δ**, in the individual ***a, b, c, r,*** r^2***, s, t,*** $2c^2$***,*** $\sqrt{t}$***, A, P, ƒ & p*** values one can see the incredible way the fundamental **ISL** number sequence — a sequence that informs the entire **BBS-ISL Matrix** — certainly comes into play here to form a consistent ***NPS*** link from and to each and every **PPT** on its **Branch**.