Table 3a

Expanded Tertiary Tree of Primitive Pythagorean Triples: UPPER *Pythagoras* BRANCH

	Trunk																														
TRUNK	PPT	*a*	*Δa*	*b*	*Δb*	*c*	*Δc*	*ΔEVEN a or b - EVEN a or b*	*(Σa+b +c)÷a= r_{next}*	*r*	*Δr*	*Δr ÷ 2*	*r^2 ÷ 4*	*$Δr^2$ ÷ 4*	*s*	*t*	*Δt*	*$2c^2$*	*t ÷ 2*	A	A ÷ 6	ΔA ÷ 6	*P*	*ΔP*	Δ ΔP	*f*	*Δf*	Δ *f*-*p*	*U ÷ c = p*	Δ p	Δp ÷ 4
1	**3-4-5**	3		4		5			4	2			1		1	2		50	1	6	1	1	12			1		0	1		
	1st-14th UPPER Tertiary Branches																														
UPPER TERTIARY BRANCH	PPT	*a*	*Δa*	*b*	*Δb*	*c*	*Δc*	*ΔEVEN a or b - EVEN a or b*	*(Σa+b +c)÷a = r_{next}*	*r*	*Δr*	*Δr ÷ 2*	*r^2 ÷ 4*	*$Δr^2$ ÷ 4*	*s*	*t = (a+c) previous*	*Δt*	*$2c^2$*	*t ÷ 2*	A	A ÷ 6	ΔA ÷ 6	*P*	*ΔP*	Δ ΔP	*f*	*Δf*	Δ *f*-*p*	*U ÷ c = p*	Δ p	Δp ÷ 4
1	**5-12-13**	5	**2**	12	**8**	13	**8**	**8**	**6**	4	**2**	1	**$4=2^2$**	**3**	1	8	**6**	336	**4**	30	5	**4**	30	18		7	**6**	**2**	**5**	**4**	**1**
2	**7-24-25**	7	**2**	24	**12**	25	**12**	**12**	**8**	6	**2**	1	**$9=3^2$**	**5**	1	18	**10**	1250	**9**	84	14	**9**	56	26	**8**	17	**10**	**4**	**13**	**8**	**2**
3	**9-40-41**	9	**2**	40	**16**	41	**16**	**16**	**10**	8	**2**	1	**$16=4^2$**	**7**	1	32	**14**	3362	**16**	180	30	**16**	90	34	**8**	31	**14**	**6**	**25**	**12**	**3**
4	**11-60-61**	11	**2**	60	**20**	61	**20**	**20**	**12**	10	**2**	1	**$25=5^2$**	**9**	1	50	**18**	7442	**25**	330	55	**25**	132	42	**8**	49	**18**	**8**	**41**	**16**	**4**
5	13-84-85	13	**2**	84	**24**	85	**24**	**24**	**14**	12	**2**	1	**$36=6^2$**	**11**	1	72	**22**	14450	**36**	546	91	**36**	182	50	**8**	71	**22**	**10**	**61**	**20**	**5**
6	15-112-113	15	**2**	112	**28**	113	**28**	**28**	**16**	14	**2**	1	**$49=7^2$**	**13**	1	96	**26**	25538	**48**	840	140	**49**	240	58	**8**	97	**26**	**12**	**85**	**24**	**6**
7	17-144-145	17	**2**	144	**32**	145	**32**	**32**	**18**	16	**2**	1	**$64=8^2$**	**15**	1	128	**30**	42050	**64**	1224	204	**64**	306	66	**8**	127	**30**	**14**	**113**	**28**	**7**
8	19-180-181	19	**2**	180	**36**	181	**36**	**36**	**20**	18	**2**	1	**$81=9^2$**	**17**	1	162	**34**	65522	**81**	1710	285	**81**	380	74	**8**	161	**34**	**16**	**145**	**32**	**8**
9	21-220-221	21	**2**	220	**40**	221	**40**	**40**	**22**	20	**2**	1	**$100=10^2$**	**19**	1	200	**38**	97682	**100**	2310	385	**100**	462	82	**8**	199	**38**	**18**	**181**	**36**	**9**
10	23-264-265	23	**2**	264	**44**	265	**44**	**44**	**24**	22	**2**	1	**$121=11^2$**	**21**	1	242	**42**	140450	**121**	3036	506	**121**	552	90	**8**	241	**42**	**20**	**221**	**40**	**10**
11	25-312-313	25	**2**	312	**48**	313	**48**	**48**	**26**	24	**2**	1	**$144=12^2$**	**23**	1	288	**46**	195938	**144**	3900	650	**144**	650	98	**8**	287	**46**	**22**	**265**	**44**	**11**
12	27-364-365	27	**2**	364	**52**	365	**52**	**52**	**28**	26	**2**	1	**$169=13^2$**	**25**	1	338	**50**	266450	**169**	4914	819	**169**	756	106	**8**	337	**50**	**24**	**313**	**48**	**12**
13	29-420-421	29	**2**	420	**56**	421	**56**	**56**	**30**	28	**2**	1	**$196=14^2$**	**27**	1	392	**54**	354482	**196**	6090	1015	**196**	870	114	**8**	391	**54**	**26**	**365**	**52**	**13**
14	31-480-481	31	**2**	480	**60**	481	**60**	**60**	**32**	30	**2**	1	**$225=15^2$**	**29**	1	450	**58**	462722	**225**	7440	1240	**225**	992	122	**8**	449	**58**	**28**	**421**	**56**	**14**
Notes	*$Σa+b+c = r_{next}$*	*$a + c = s_{next}$ $a= r + s$ $a = c - t$*	*a↑ = 2*	*$b + c = t_{next}$ $b = r + t$ $b = c - s$*	*b↑ = 4 + ∑Δ*	*$c = p_{next}$ $c = 2r + p$ $c = r + s + t$ $c= a + t$ $c= b + t$ $c= a + b - r$ $c= U/p$*	*c↑ = 4 + ∑Δ*	Δ of EVEN *a* or *b* from the NEXT EVEN *a* or *b* value = 4 + ∑Δ	*$P=Σa+b+c = (4A)/r = 2c + r$ $P/a = r_{next}$*	The *$Δr = Σs + t_{next} = Σs_{next} + t$* The *$Δr/2 = 5—29—169—…$* and = the *p* & *ΔP/2* *$r=√2st$* *$r= (a + b) - c.$* *$r↑ = 2$*				Dividing *r^2* by 4 is the same as ÷ it by the *r^2* of 3-4-5. The Δ in *r^2*/4 also generates the same 5—29—169…sequence.		*$r^2/2 = st.$* *$s_{next} = a + c$* *$t_{next} = b + c$* *s* = constant *1* *$t↑ = 4+ Σ\Delta$* and when ÷ by *2 = 1—4—9—16—….* sequence that defines the BBS-ISL MATRIX (BIM)				The ∑ of A/6 - $A_{next}/6 = r^2/4_{next}$ *$A = (Pr)/4 = bh/2$* When *$ΔA ÷ 6 = 1—4—9—16—…. =$* sequence that defines the BBS-ISL MATRIX (BIM)			*$P=Σa+b+c = (4A)/r = 2c + r$* *$P/a = r_{next}$* The *Δ* in *P* is the accumulated *$Σ\Delta + 8$*			The Δ in *f* is the accumulated ∑Δ + 4 The Δ between *f* and *p* is the accumulated ∑Δ+ 2		*$p = c - 2r$* *$p_{next} = c$* *$Δp ÷ 4 = r_{previous}/2$* On the matrix, a 45° diagonal from *p* on the AXIS is a COMMON DIAGONAL to all three TERTIARY MEMBERS of a given Branch. This unites them.		
Summary	Finding the NEXT UPPER is easy! Since we know: *a↑=2 b↑=∑Δ+4* and *c↑=∑Δ+4* , one can simply add these differences, Δ, to the previous to generate the NEXT, e.i. to **5—12—13** add 2 to *a*=7, *add* 8+4=8 to *b*=24 & 8+4=12 to *c*=25 to give the **7—24—25** NEXT UPPER PPT. Because *$(∑a+b+c)/a = ∑/a = r_{next} = *P/a$,* we have for the **5—12—13** : (5+12+13)/5 = 30/5 = 6 = *$r_{next} = P/a$,* with *P*=30. We also know that *r*↑ by 2, so *r* + 2 = 4 + 2 = 6 = *r_{next}*. *Then $r^2/2 = st = 6^2/2 = 18$.* Knowing *s*=constant 1, therefore *t_{next}* = 18, confirmed by knowing that *t_{next}*/2 = the 1-4-9-16-… ISL sequence, and that *$a + c = t_{next} = 5 + 13 = 18$*, we can easily calculate *a, b* & *c*: *$a= r + s = 6 + 1 = 7$* *$b= r + t = 6 + 18 = 24$* *$c= r + s + t = 6 + 1 + 18 = 25$* to give the **7—24—25** NEXT UPPER PPT. As *P*↑ by the ∑Δ + 8, we add 18 + 8 = 26 to 30 = 56, confirming *$P = a + b + c = 7 + 24 + 25 = 56$*, with the starting *P* of the **5—12—13,** *$P = 2c + r = (2x13) + 4 = 30$* & *$*P/a = r_{next} = 30/5 = 6$* = the *r*-value of the **7—24—25 PPT.** From *$f = t - s = b - a = 18 - 1 = 24 - 7 = 17$*, we also know *f*↑ by the ∑Δ + 4 sequence, as does the *p*↑. *$c=p_{next}$* is yet another confirming pattern, as the *c* = 13 of the **5—12—13** = the *$p_{next,}$ i.e. the p=13 of* the **7—24—25 PPT. All Tertiary Branch Clusters have the same *p*-value!**																																
Table 3a	Key: **PPT=Primitive Pythagorean Triple**; ***r*=even #** such that ***$r^2/2=st$*** where ***s,t*** are Factor Pairs; **A=Area; 4A=4Area; 8A=8Area;** ***f=b-a=t-s*** & ***$f^2=(b-a)^2$,*** as ***$a^2 + b^2 = c^2 = 4A + f^2 = (8A + f^2) - 4A$*** & ***U/c=p. $U=s^2+t^2$ A=Pr/4*** & ***P=4A/r=2c+r*** , whereas ***c=2r+p***. ***The Tree of Pythagorean Triples*** branches from the **3-4-5 PPT Trunk** first into a 3-part main branch, each of which further branches into 2nd, 3rd, 4th, …, **Tertiary Branches**. Each **Tertiary** follows the lead ***f***-value of its predecessor, but is actually formed as an intermediary to the Upper and Lower branches of which it is a part. All **PPT**s — with no repeats — are to be found. ***Pythagoras*** first discovered the **UPPER Branch** sequence, ***Plato*** (a century later) discovered the **LOWER Branch** sequence. The **MIDDLE Branch** sequence follows as an intermediary, hybrid sequence of the **UPPER** and **LOWER**, plus some amazing ***Number Pattern Sequences*** (***NPS***) all to itself. Using the ***Expanded Dickson Method*** on the **BBS-ISL Matrix**, every **PPT Branch** is accounted for by the previous **Branch**. This is done by enlisting the ***r,s,t,A,4A,8A,f*** associated values as seen in the **Table 2** series . All these values are derived directly from the respective **PTT** by both algebra and geometry. Now, in **Table 3a,** we look at the overall ***NPS*** of just one **Branch** sequence: here we are looking at the **UPPER *Pythagoras* Branches**. By parsing out the differences, **Δ**, in the individual ***a, b, c, r, r^2, s, t, $2c^2$, $√^2$, A, P, f & p*** values one can see the incredible way the fundamental **ISL** number sequence — a sequence that informs the entire **BBS-ISL Matrix** — certainly comes into play here to form a consistent ***NPS*** link from and to each and every **PPT** on its **Branch**.																																