Table 2a

# Tertiary Tree of Primitive Pythagorean Triples

Key: PPT=Primitive Pythagorean Triple; $r$=even # such that $r^2/2=st$ where $s,t$ are Factor Pairs; A=Area; 4A=4Area; 8A=8Area; $f$=b-a & $f^2=(b-a)^2$, as $a^2 + b^2 = c^2 = 4A + f^2 = (8A + f^2) - 4A$

| Trunk | | | | | | | | A÷7 | 1st Tertiary Branch | | | | | | | | A÷7 | 2nd Tertiary Branches | | | | | | | | A÷7 | 3rd Tertiary Branches | | | | | | | | A÷7 |
| PPT | r | s | t | A | 4A | 8A | f | | PPT | r | s | t | A | 4A | 8A | f | | PPT | r | s | t | A | 4A | 8A | f | | PPT | r | s | t | A | 4A | 8A | f | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3-4-5 | 2 | 1 | 2 | 6 | 24 | 48 | 1 | | 5-12-13 | 4 | 1 | 8 | 30 | 120 | 240 | 7 | | 7-24-25 | 6 | 1 | 18 | 84 | 336 | 672 | 17 | ✓ | 9-40-41 | 8 | 1 | 32 | 180 | 720 | 1440 | 31 | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | 88-105-137 | 56 | 32 | 49 | 4620 | 18480 | 36960 | 17 | ✓ |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | 60-91-109 | 42 | 18 | 49 | 2730 | 10920 | 21840 | 31 | ✓ |
| | | | | | | | | | | | | | | | | | | 48-55-73 | 30 | 18 | 25 | 1320 | 5280 | 10560 | 7 | | 105-208-233 | 80 | 25 | 128 | 10920 | 43680 | 87360 | 103 | ✓ |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | 297-304-425 | 176 | 121 | 128 | 45144 | 180576 | 361152 | 7 | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | 84-187-205 | 66 | 18 | 121 | 7,854 | 31,416 | 62,832 | 103 | ✓ |
| | | | | | | | | | | | | | | | | | | 28-45-53 | 20 | 8 | 25 | 630 | 2520 | 5040 | 17 | ✓ | 95-168-193 | 70 | 25 | 98 | 7980 | 31920 | 63840 | 73 | ✓ |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | 207-224-305 | 126 | 81 | 98 | 23184 | 92736 | 185472 | 17 | ✓ |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | 44-117-125 | 36 | 8 | 81 | 2574 | 10296 | 20592 | 73 | |
| | | | | | | | | | 20-21-29 | 12 | 8 | 9 | 210 | 840 | 1680 | 1 | ✓ | 39-80-89 | 30 | 9 | 50 | 1560 | 6240 | 12480 | 41 | | 57-176-185 | 48 | 9 | 128 | 5016 | 20,064 | 40,128 | 119 | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | 336-377-505 | 208 | 128 | 169 | 63,336 | 253,344 | 506,688 | 41 | ✓ |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | 180-299-349 | 130 | 50 | 169 | 26,910 | 107,640 | 215,280 | 119 | |
| | | | | | | | | | | | | | | | | | | 119-120-169 | 70 | 49 | 50 | 7140 | 28560 | 57120 | 1 | ✓ | 217-456-505 | 168 | 49 | 288 | 49,476 | 197,904 | 395,808 | 239 | ✓ |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | 696-697-985 | 408 | 288 | 289 | 242556 | 970224 | 1940448 | 1 | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | 220-459-509 | 170 | 50 | 289 | 50,490 | 201,960 | 403,920 | 239 | |
| | | | | | | | | | | | | | | | | | | 36-77-85 | 28 | 8 | 49 | 1386 | 5544 | 11088 | 41 | ✓ | 175-288-337 | 126 | 49 | 162 | 25,200 | 100,800 | 201,600 | 113 | ✓ |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | 319-360-481 | 198 | 121 | 162 | 57,420 | 229,680 | 459,360 | 41 | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | 52-165-173 | 44 | 8 | 121 | 4,290 | 17,160 | 34,320 | 113 | |
| | | | | | | | | | 8-15-17 | 6 | 2 | 9 | 60 | 240 | 480 | 7 | | 33-56-65 | 24 | 9 | 32 | 924 | 3696 | 7392 | 23 | ✓ | 51-140-149 | 42 | 9 | 98 | 3,570 | 14,280 | 28,560 | 89 | ✓ |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | 252-275-373 | 154 | 98 | 121 | 34,650 | 138,600 | 277,200 | 23 | ✓ |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | 120-209-241 | 88 | 32 | 121 | 12,540 | 50,160 | 100,320 | 89 | |
| | | | | | | | | | | | | | | | | | | 65-72-97 | 40 | 25 | 32 | 2340 | 9360 | 18720 | 7 | | 115-252-277 | 90 | 25 | 162 | 14,490 | 57,960 | 115,920 | 137 | ✓ |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | 396-403-565 | 234 | 162 | 169 | 79,794 | 319,176 | 638,352 | 7 | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | 136-273-305 | 104 | 32 | 169 | 18,564 | 74,256 | 148,512 | 137 | ✓ |
| | | | | | | | | | | | | | | | | | | 12-35-37 | 10 | 2 | 25 | 210 | 840 | 1680 | 23 | ✓ | 85-132-157 | 60 | 25 | 72 | 5,610 | 22,440 | 44,880 | 47 | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | 133-156-205 | 84 | 49 | 72 | 8,814 | 35,256 | 70,512 | 23 | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | 16-63-65 | 14 | 2 | 49 | 504 | 2,016 | 4,032 | 47 | ✓ |

Key: PPT=Primitive Pythagorean Triple; $r$=even # such that $r^2/2=st$ where $s,t$ are Factor Pairs; A=Area; 4A=4Area; 8A=8Area; $f$=b-a & $f^2=(b-a)^2$, as $a^2 + b^2 = c^2 = 4A + f^2 = (8A + f^2) - 4A$

**The Tree of Pythagorean Triples** branches from the **3-4-5 PPT** trunk first into a 3-part main branch, each of which further branches into 2nd, 3rd, 4th, …, tertiary branches. Each tertiary follows the lead $f$-value of its predecessor, but is actually formed as an intermediary to the upper and lower branches of which it is a part. All **PPT**s — with no repeats — are to be found. **Pythagoras** first discovered the UPPER branch sequence, **Plato** (a century later) discovered the LOWER branch sequence. The MIDDLE branch sequence follows as an intermediary, hybrid sequence of the UPPER and LOWER.

Using the *Expanded Dickson Method* on the **BBS-ISL Matrix**, every **PPT** branch is accounted for by the previous branch. This is done by enlisting the $r,s,t$,A,4A,8A,$f$ associated values. All these values are derived directly from the respective **PTT** by both algebra and geometry. In **Table 2a** we looked at the overall.    In **Table 2b**, we examine how the UPPER and LOWER branches (blue) are made from the trunk (red).    In **Table 2c**, we see how the MIDDLE branch (red) is formed from the UPPER and LOWER (blue) branches and the trunk (red). As a fractal, this **Number Pattern Sequence** that defines the first branchings, continues through the entire tree.    **Table 2d** shows BLUE branching to **2nd Tertiary Branches**.  **Table 2e** reveals the power of $f$.    **Table 2f** tells all.

Table 2a